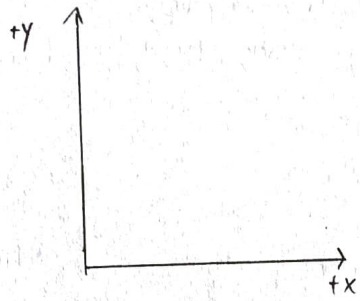
Lecture 1: Cartesian Coordinates and 3D space

Coordinate Axes

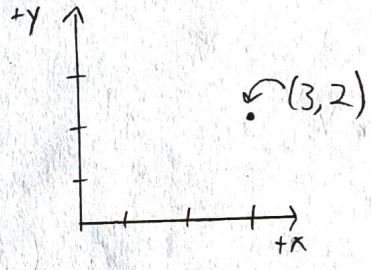
In 2D space we use the familiar x- and y-axes.



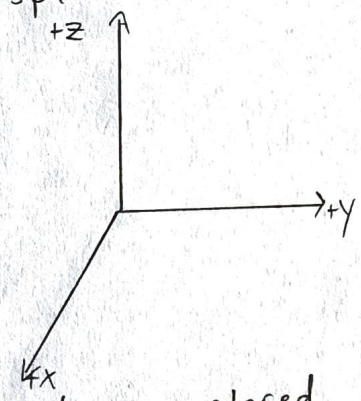
You are likely familiar with coordinate pairs on this Cartesian Plane. For example:

Ex 1. Plot the point $(3, 2)$.

In an ordered pair, the first number is the x position and the second is the y. So,

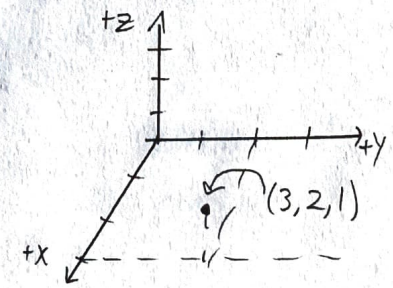


In 3-space we add a z-axis:



We now have ordered triples of the form (x, y, z).

Ex. 2 Plot the point $P = (3, 2, 1)$



Since any point in this space can be specified by 3 numbers, we call it 3 dimensional.

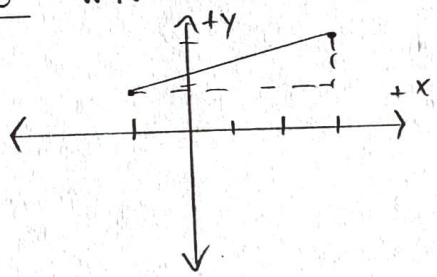
Coordinate Planes & Octants

These 3 coordinate axes also specify 3 coordinate planes: the xy-, xz-, and yz-planes. These coordinate planes also divide space into 8 octants. Only the octant where every coordinate is positive, (+,+,+), has a name. It is the First Octant.

Distance

We will use the "Euclidean Metric" to define distance. This is the common meaning of "distance".

Ex. 3 What is the distance between $P=(3,2)$ and $Q=(-1,1)$?



$$|PQ|^2 = (3-{-1})^2 + (2-1)^2$$
$$|PQ|^2 = 4^2 + 1^2$$
$$|PQ| = \sqrt{17}$$

In general in 2D the distance between $P=(x_0,y_0)$ and $Q=(x_1,y_1)$ is $|PQ| = \sqrt{(x_0-x_1)^2 + (y_0-y_1)^2}$

The extension to 3D is simple. With $P=(x_0,y_0,z_0)$ and $Q=(x_1,y_1,z_1)$,

$$|PQ| = \sqrt{(x_0-x_1)^2 + (y_0-y_1)^2 + (z_0-z_1)^2}$$

Ex. 4 What is the distance between $P=(3,2,-1)$ & $Q=(-5,4,-1)$

$$|PQ| = \sqrt{(3-{-5})^2 + (2-4)^2 + (-1-{-1})^2}$$
$$|PQ| = \sqrt{8^2 + (-2)^2 + 0^2}$$
$$|PQ| = \sqrt{68}$$

Metrics are defined by these three properties:

i) $|PQ| = 0$ iff $P=Q$ — Identity

ii) $|PQ| = |QP|$ — Symmetry

iii) $|PQ| \leq |PR| + |RQ| \quad \forall R$ — Triangle Inequality

Spheres and Balls

A sphere centered at $P_0 = (x_0, y_0, z_0)$ with radius a is described by the equation: $(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = a^2$

A ball centered at P_0 with radius a is given by:

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 \leq a^2$$

Ex. 5 Show that $x^2+y^2+z^2+4x-6y+2z+6=0$ is a sphere and find its center and radius.

$$x^2+4x+y^2-6y+z^2+2z+6=0$$

$(x+2)^2 = x^2+4x+4$ complete the square

$(y+3)^2 = y^2+6y+9$

$(z+1)^2 = z^2+2z+1$

$$(x+2)^2-4+(y+3)^2-9+(z+1)^2-1+6=0$$

$$(x+2)^2+(y-3)^2+(z+1)^2=8 \qquad P_0=(-2,3,-1) \qquad a=\sqrt{8}$$

Cylinders

A cylinder centered at P_0 with radius a is given by

$$(x-x_0)^2+(y-y_0)^2=a^2$$